MONO LAKE – POSSIBLE FUTURES

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EXTENDED ABSTRACT

Mono Lake is a hypersaline lake, having approximately 7.3 % salt content, and consisting mainly of sodium salts of chlorides, carbonates, and sulfates. The lake is situated in eastern California, USA, and is fed by five mountain streams. The lake has an area of ~ 182 km^2, a volume of ~ 3.22 km^3, and a maximum depth of 48.8 m. The lake had a salinity of 80 gm(s)/L in the year 2002. Additionally Mono Lake is ~ 760,000 years old, one of the oldest in North America. In 1994 in the attempt to reverse the desiccation of the lake, the California State Water Resources Board issued D 1631. It concerned the flows and maintenance of the
future of the lake. [1], [2], [3], [4]
This paper considers three possible futures:

I. A Salinity Source For PRO, Pressure Retarded Osmosis
II. Developing A Small Fresh Water Lake
III. A Lake Complying With The Water Resources Board

In Number I, several forms of the van't Hoff equation are compared for the pressure P, in atmospheres, using the Mono Lake salinity, and the "sea water" salinity.
In Number II, equations for the mass m, gm(t)*af/L, the volume V, af, and salt content Z, gm(s)*af/L are developed. Then, for assumed initial conditions, solutions are given in a table and a graph.
In Number III, Initial flow conditions are given that will reverse the desiccation and satisfy the 20 year Water Resources Board plan.

M = mass, gm(t)*af/L, V = volume, af,
Z = salt content, gm(s)*af/L, G = salinity, gm(s),
R = density, gm(t)/L, S = % salt, gm(s)/100 gm(t).

I. A Salinity Power Source for PRO, Pressure Retarded Osmosis

A comparison is developed, in atmospheres, between the pressure developed from the salinity of "sea water", and the pressure developed from the salinity of Mono Lake.
Several forms of the van't Hoff equation are used.[5], [6], [7]
Osmotic Pressure

$$\hat{}P = Ps-Pf \tag{1}$$

Where

$$P = MR`T = (G/Mw)*R`T \tag{2}$$

And so

$$Pf = (Gf/Mw)*R'T \tag{3}$$

For fresh water, see Section IIb,

$$Gf= 0.064 \text{ gm(s)/L}$$
$$Mw \sim 80 \text{ gm(s)/mol}$$

So

$$Pf = (0.064/80)*24.618 = 0.01969 \text{ atm} \sim 0 \tag{4}$$

Thus

^P = Ps = P (5)

In the above

^P = osmotic Pressure, atm R` = 0.08206 L* atm/mol* K
Ps = P salt solution, atm T = 300 K
Pf = P fresh water, atm RT = 24.618 L* atm/mol
M = mol/L, molarity Mw = gm(s)/mol
P = Pressure, atm G = gm(s)/L

Now from references [8] and [9], for "sea water" ~ 85% NaCl,

Table I

P, bar	G	M	a		i	Mw	H	kwh/m^3	
27	--	--	--	--	--	270	0.7		
20.3	35	0.6	0.676	2		58.5		203	0.563

Where

i = ions per molecule, dimensionless
a = coefficient of activity, dimensionless
H = column height of water, m
1 atm = 1.0135* bar

Now consider

P = MR`T (6)
P = iaMR`T (7)
P = iMR`T = RT`*Sum Mj (8)

Thus from [8]

P = MR`T = (0.6)*24.618 = 14.7708 atm (6a)
P = iaMR`T = 2*(0.676)*(0,6)*24.618 = 19.970 atm (7a)
P = iMR`T = 2*(0.6)*24.618 = 29.5416 atm (8a)

From [7]

P = 26.66 atm (8b)

From the "sea water" analysis, we can see that equations (6a) and (8a) provide the approximate lower and upper marginal boundaries, for the estimation of the pressure P, in atmospheres.

I. A Salinity Source For PRO, Pressure Retarded Osmosis

Now from references [9] and [14]

Table II

m	Sum mj	G	Rho	T(assumed)
1.09	2.06	90	1071.5	300

Now

M = Rho*m/1000 = gm(t)/L*(mol/kg(t))*(kg(t)/gm(t)) = mol/L (9)

And

Sum mj = mt, (10)
Mt = Rho*mt/1000 = Sum Mj = Rho* Sum mj/1000 (11)

Where

m = mol/1000 gm(t) R' = 0.08206 L atm/mol K
Rho = gm(t)//L T = 300 K
M = mol/L R'T = 24.618 L atm/mol

Thus

M = Rho*m/1000 = 1071.5*1.09/1000 = 1.16795 mol/L (12)
Sum M = Rho*Sum mj = 1075.1*2.06/1000 = 2.2073 mol/L (13)

Thus

P = MR'T =1.16795*24.618 = 28.752 atm (6b)

$$P = R'T*Sum\ Mj = 24.618*2.2073 = 54.339 \text{ atm} \quad (8b)$$

Now

$$Ch = 10.333 \text{ m/atm}$$
$$Cw = 0.028146 \text{ kwh/m3 atm}$$

So

$$H = Ch*P, \text{ m} \quad (14)$$
$$kwh/m^3 = Cw*P \quad (15)$$
$$kwh/af = kwh/m^3*m^3/af=1233.47*Cw*P \quad (16)$$
$$kwh/y = kwh/af*af/y = 1223.47*Cw*P*V' \quad (17)$$
$$kw = kwh/y*y/h = 1233.47*Cw*P*V'/8760 \quad (18)$$

Thus for P = 28.752 atm, the lower boundary for Mono Lake, with V' = 10^5 af/y, results in

$$H = 297.094 \text{ m} \quad (14a)$$
$$kwh/m^3 = 0.80925 \text{ kwh/m}^3 \quad (15a)$$
$$kwh/af = 998.190 \text{ kwh/af} \quad (16a)$$
$$kwh/y = 99.819*10^6 \text{ kwh/y} \quad (17a)$$
$$kw = 11{,}394 \text{ kw} \quad (18a)$$

For P = 54.339, the upper boundary for Mono Lake, and V' = 10^5 af/y, results in

$$H = 561.484 \text{ m} \quad (14b)$$
$$kwh/m^3 = 1.5294 \text{ kwh/m}^3 \quad (15b)$$
$$kwh/af = 1866.50 \text{ kwh/af} \quad (16b)$$
$$kwh/y = 188.65*10^6 \text{ kwh/y} \quad (17b)$$
$$kw = 21{,}535 \text{ kw} \quad (18b)$$

These results provide the approximate lower and upper bounds, when G = 90 gm(s)/L, for Mono Lake.

I. A Salinity Source For PRO, Pressure Retarded Osmosis

Now Table III contains, for the lower boundary, some necessary conditions, in order to extend
the analysis.

Table III

P, atm	G, gm(s)/L	R'T, L atm/mol K
28.752	90	24.618

Now

$$P = MR'T = (G/Mw)*R'T \quad (19)$$

Thus

$$Mw = G*R'T/P = 90*24.618/28.752 = 77.0597 \text{ gm(s)/mol} \quad (20)$$
$$= \text{mixture molecular weight}$$

So

$$P = G*R'T/Mw = G*24.618/77.0597 = 0.31946*G \quad (21)$$
$$P(G) = G/3.1302, \text{ atm} \quad (22)$$

In reference [14], for Mono Lake

$$36 < G < 698, \text{ gm(s)/L}$$

Thus, if G = 313.02 gm(s)/L, then

$$P = 100 \text{ atm} \quad (23)$$
$$H = 1033.3 \text{ m} \quad (24)$$
$$kwh/m^3 = 2.8146 \text{ kwh/m}^3 \quad (25)$$

These results indicate the future potential of Mono Lake. The characteristics values of Mono Lake exceed the characteristic values of "sea water", for developing osmotic power, PRO. This should result, in serious consideration, for a Mono Lake PRO site, in the future.

IIa. Developing A Small Fresh Water Lake

Controlling Equations [10]

Mass, M(t)

$$M(t) = R(t)*V(t) = \text{Sum } Mj(t) \quad (1a)$$

$$M'(t) = R(t)*V'(t)+R'(t)*V(t) \quad (1b)$$

Where

$R(t) = gm(t)/L$, density

$V(t) = af$, volume

$M(t) = gm(t)*af/L$

Also

$$M'(t)= \text{Sum } Mj'(t) = Min'(t)+Mra'(t)+Msp'(t)+Mev'(t)+Mse'(t)+Mex'(t) \quad (1c)$$

Where

$Min'(t) = Rin*Vin'(t)$

$Mra''(t) = Ro*Vra'(t)$

$Msp'(t) = Rsp*Vsp'(t)$

$Mev'(t) = - Ro*Vev'(t)$

$Mse'(t) = - R(t)*Vse'(t)$

$Mex'(t) = - R(t)*Vex'(t)$

Thus

$$\begin{aligned} M'(t) &= R(t)*V'(t) + R'(t)*V(t) \\ &= Rin*Vin'(t) + Ro*Vra'(t) + Rsp*Vsp'(t) - Ro*Vev'(t) \\ &\quad - R(t)*[Vse'(t) + Vex'(t)] \end{aligned} \quad (2)$$

Volume, V(t)

$$\begin{aligned} V'(t) &= \{Rin*Vin'(t) + Ro*Vra'(t) + Rsp*Vsp'(t) - Ro*Vev'(t) - R'(t)*V(t)\}/R(t) \\ &\quad - Vsp'(t) - Vex'(t), \qquad V(0) = Vo \end{aligned} \quad (3)$$

Salt, Z(t)

$$Z(t)=G(t)*V(t) = \text{Sum } Zj(t) \quad (4)$$

Where

$G(t) = gm(s)/L$, salinity

$Z(t) = gm(s)*af/L$

Also

$$Z'(t) = Zin'(t)+Zra'(t)+Zsp'(t)+Zev'(t)+Zse'(t)+Zex'(t) \quad (5)$$

Where

$Zin'(t) = Gin*Vin'(t)$

$Zra'(t) = 0 = Zev'(t)$

$Zsp'(t) = Gsp*Vsp'(t)$

$Zse'(t) = - G(t)*Vse'(t)$

$Zex'(t) = - G(t)*Vex'(t)$

Then

$$Z'(t) = Gin*Vin'(t) + Gsp*Vsp'(t) - G(t)*[Vse'(t) + Vex'(t)], \quad Z(0) = Zo \quad (6)$$

Salinity, G(t)

$$G(t) = Z(t)/V(t), \ gm(s)/L \quad (7)$$

Density, R(t)

$$R(t) = F(t), \ gm(t)/L \quad (8)$$

%Salt, S(t)

$$S(t) = 100*G(t)/R(t), \ gm(s)/100*gm(t) = \%Salt \quad (9)$$

Water, W(t)

$$W(t) = M(t) - Z(t), \ gm(w)*af/L \quad (10a)$$

$$w(t) = R(t) - G(t), \ gm(w)/L \quad (10b)$$

II.b Developing A Small Fresh Water Lake

Volume, V(t)

V'(t) = {Rin*Vin'(t)+Ro*Vra'(t)+Rsp*Vsp'(t)-Ro*Vev'(t)-R'(t)*V(t)}/R(t)
-Vse'(t)-Vex'(t), V(0) = Vo (1)

Given Parameters, [11]

Vot = 2.7*10^6 af	Elev = 6385 ft, surface
Aot = 45,900 a	Ro = 1,000 gm(t)/L
ao = bo= 0.1	R(t) = Ro + E*exp(-g*t)
Vo = ao*Vot = 0.27*10^6 af	E = 62 gm(t)/L*y
Ao = bo*Aot = 4,590 a	g = 0.5362 1/y
Hra = 0.7 ft/y	Vin'(t)ot = 152,496 af/y
Hev = 3.75 ft/y	Vsp'(t)ot = 48,000 af/y
Go = 78 gm(s)/L	Vse'(t)ot = 34,000 af/y

R'(t)= - g*E*exp(-g*t) gm(t)/L*y

If V'(t) = 0, V(t) = Vo, A(V) = Ao, Ro ~ Rin ~ Rsp, then

Vex'(t) = [Ro/R(t)]*[Vin'(t)+Vra'(t)+Vsp'(t)-Vev'(t)-R'(t)*V(t)/Ro] –Vse'(t) (2)
V(0) = Vo

Where

Vin'(t) = Vin'(t)ot = 152,496 af/y = Vin'
Vra'(t) = bo*Hra*Ao=0.1*0.7*45,900 = 3,213 af/y = Vra'
Vsp'(t) = bo*Vsp'(t)ot = 0.1*48,000 = 4,800 af/y = Vsp'
Vev'(t) = bo*Hev*Ao = 0.1*3.75*45,900 = 17,212.5 af/y = Vev'
Vse'(t) = bo*Vse'(t)ot = 0.1*34,000 = 3,400 af/y = Vse'
R'(t)*Vo/Ro = -g*E*exp(-g*t)*Vo/Ro = -0.5362*62*exp(-G*t)/1000
= - 8,9875.9*E*exp(-g*t) af/y

Thus

Vex'(t) = [143,296.5 + 8975.9*exp(-g*t)]/[1.000 + 0.062*exp(-g*t)] - 3,400 (3)

Where

Vex'(0) = 139,296.5 af/y
Vex'(inf) = 139,896.5 af/y

Also

Vex'(t) + Vse'(t) = [143,296 + 8,975.9*exp(-g*t)]/[1.000 + 0.062*exp(-g*t)] (4)

Salt, Z(t)

Z(t) = G(t)*V(t) = Sum Zn(t) (5a)
Z'(t) = Zin'(t) + Zra'(t) + Zsp'(t) + Zev'(t) + Zse'(t) + Zex'(t) (5b)

Given Parameters

Gin = 0.064 gm(s)/L	Gsp = Gin gm(s)/L
Gse(t) = G(t) gm(s)/L	Gex(t) = G(t) gm(s)/L
Gra = 0	Gev = 0
Go = 78 gm(s)/L	Zo = Go*Vo

Now

Zin'(t) = Gin*Vin = 0.064*152496 = 8,759.9 gm(s)*af/L(y
Zra'(t) = 0 = Zev'(t)
Zsp(t) = Gsp*Vsp'= 0.064*4800 = 307.2 gm(s)*af/L*y
Zse'(t) = - G(t)*Vse'(t) = - Z(t)*3300/V(t)
= - 0.0121592 *Z(t) gm(s)*af/L*y
Zex'(t) = - G(t)*Vex'(t) gm(s)*af/L*y

So

Z'(t) = Gin*Vin' + Gsp*Vsp'- G(t)*[Vse'(t) + Vex'(t)], Z(0) = Zo (6)

Thus

$$Z'(t) = 10{,}066.9 + [0.530727 + 0.033244*\exp(-g*t)]*Z(t)/[1.000 + 0.062*\exp(-g*t)]$$

$$Z(0) = Zo = 21.06*10^6 \text{ gm(s)*af/L} \quad (7)$$

A numerical solution to equation (7) is given in Table IV, in the following page.

Now

$$Z(t) = \text{gm(s)*af/L} \quad (7a)$$

and

$$\text{gm(s)*af/L*(L/m^3)*(m^3/af)} = \text{gm(s)} = \text{(L/m^3)*(m^3/ af)*}Z(t) \quad (7b)$$

Thus

$$1.23347*10^6*Z(t) = \text{gm(s)} \quad (7c)$$

Salinity, G(t)

$$G(t) = Z(t)/V(t) = Z(t)/Vo = Z(t)/(0.27*10^6) \text{ gm(s)/L} \quad (8)$$

So

$$G(t) = 78*\exp(-g*t) + 0.0703*[1 - \exp(-g*t)] \text{ gm(s)/L} \quad (9)$$

$G(0) = 78$ gm(s)/L

$G(inf) = 0.0703$ gm(s)/L

$g = 0.5362$ 1/y

Density, R(t)

$$R(t) = 1000 + 62*\exp(-g*t) \text{ gm(t)/L} \quad (10)$$

$R(0) = 1062$ gm(t)/L

$R(inf) = 1000$ gm(t)/L

%Salt, S(t)

$$S(t) = \text{gm(s)/L*L/gm(t)*gm(t)/100gm(t) \%Salt} \quad (11a)$$

$$S(t) = 100*G(t)/R(t) \text{ \%Salt} \quad (11b)$$

$$S(t) = 100*[78*\exp(-g*t) + 0.0703*(1 - \exp(-g*t)]/[1.000 + 62*\exp(-g*t)] \quad (11c)$$

$S(0) = 7.345$ %Salt

$S(inf) = 0.00703$ %Salt

Water, W(t)

$$W(t) = M(t) - Z(t) = w(t)*V(t) \text{ gm(w)*af/L} \quad (12)$$

So

$$w(t) = R(t) - G(t) \text{ gm(w)/L} \quad (13)$$

$$w(t) = 999.9297-15.9297*\exp(-g*t) \text{ gm(w)/L} \quad (14)$$

$w(0) = 984$ gm(w)/L

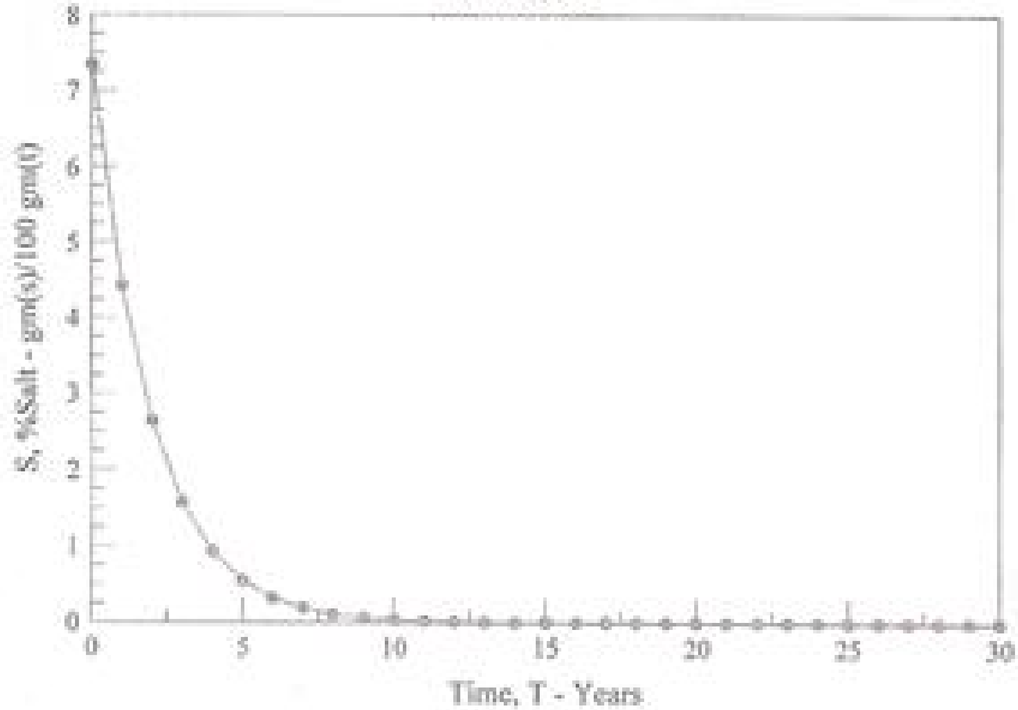
$w(inf) = 999.9297$ gm(w)/L

The solutions for G(t), R(t), S(t) and w(t) are also shown in Table IV.

Sheet: C:\Program Files\PSI\PSIPLOT\samk.pdw

	T,year	Z/10^6	S,gm(s)/L	R,gm(l)/L	S, %salt	w,gm(w)/L	Table IV
1	0.000000	21.0000	79.0000	1052.0000	7.3448	984.0000	
2	1.000000	12.3017	46.6000	1036.3387	4.4283	992.0035	
3	2.000000	7.2662	27.9164	1021.2680	2.6404	994.5454	
4	3.000000	4.2903	16.0137	1012.8482	1.5723	996.0204	
5	4.000000	2.5339	9.2421	1007.2480	0.9323	997.9710	
6	5.000000	1.4955	5.5539	1004.2339	0.5529	998.7708	
7	6.000000	0.8898	3.2955	1002.0364	0.3287	999.2424	
8	7.000000	0.5311	1.9679	1001.4062	0.1964	998.8203	
9	8.000000	0.3302	1.1656	1000.6740	0.1165	999.4808	
10	9.000000	0.1881	0.7594	1000.5136	0.0720	999.7572	
11	10.000000	0.1232	0.4562	1000.2018	0.0488	999.8433	
12	11.000000	0.0800	0.2973	1000.1770	0.0297	999.8798	
13	12.000000	0.0600	0.2039	1000.1039	0.0204	999.9002	
14	13.000000	0.0402	0.1500	1000.0916	0.0149	999.9122	
15	14.000000	0.0314	0.1164	1000.0358	0.0116	999.9134	
16	15.000000	0.0252	0.0974	1000.0210	0.0097	999.9226	
17	16.000000	0.0233	0.0860	1000.0133	0.0086	999.0201	
18	17.000000	0.0215	0.0797	1000.0072	0.0080	999.0276	
19	18.000000	0.0205	0.0759	1000.0042	0.0076	999.0255	
20	19.000000	0.0198	0.0736	1000.0020	0.0074	999.0246	
21	20.000000	0.0195	0.0727	1000.0015	0.0073	999.0253	
22	21.000000	0.0193	0.0714	1000.0009	0.0071	999.0240	
23	22.000000	0.0191	0.0739	1000.0005	0.0071	999.0239	
24	23.000000	0.0191	0.0708	1000.0003	0.0071	999.0257	
25	24.000000	0.0190	0.0705	1000.0002	0.0070	999.0257	
26	25.000000	0.0190	0.0704	1000.0001	0.0070	999.0257	
27	26.000000	0.0190	0.0703	1000.0001	0.0070	999.0257	
28	27.000000	0.0190	0.0703	1000.0000	0.0070	999.0257	
29	28.000000	0.0190	0.0703	1000.0000	0.0070	999.0257	
30	29.000000	0.0190	0.0703	1000.0000	0.0070	999.0257	
31	30.000000	0.0190	0.0703	1000.0000	0.0070	999.0257	

Figure 1
Mono Lake
S, % Salt, vs Time
naml 100c



III. A Lake Complying With The Water Resources Board
From references [12] and [13],

Table V

Year	Area, a	Volume, af	Salinity, gm(s)/L	Density, gm(t)/L
1999	45,900	2.7*10^6	78	1062
2019	48,960	3.0*10^6	69.3	1056
1963	48,739	3.037*10^6	69	1056

Historically, as late as 1963, the Water Board requirements for the future were met in 1963 and for many previous years. [11] Using the conditions listed for the year 1999, in the equations for volume, salt, salinity, density and water content, as developed in Section IIb, and using the conditions listed in the parameters given below:

Parameters

Vin'(t) = 144943 af/y	Vra'(t) = Hra*A(V)
Vsp'(t) = 48,000 af/y	Vev'(t) = Hev*A(V)
Vsp'(t) = 34,000*A(V)/Ao	Vex'(t) = 0
Hra = 0.7 ft/y	Hev = 3.75 ft/y

The results are close to the values listed for 2019.

CONCLUSION

The three possible futures for Mono Lake have been considered herein. It is seen that the Mono Lake PRO prospects are greater than the “sea water” PRO prospects. Additionally a small fresh water lake is possible in about 10 years. Satisfying the State Water Resources Board conditions was possible in the near past, and a possible future result is suggested.

Nomenclature

A= area, acre, a
af = acre-feet
ao = volume coefficient, dimensionless
Ao, Aot = starting areas, a
bo = area coefficient, dimensionless
Ch = water column factor, m/atm
Cw = work/(m^3*atm)
E = density coefficient, gm(t)/L*atm
G = salinity, gm(s)/L
H = water column height, m
Hev = evaporation factor, ft/y
Hra = rain factor, ft/y
i = ions/molecule, dimensionless
L = liter
M = molarity, mol/L
M(t) = gm(t)*af/L
m = molality, mol/kg(t)
P = pressure, atm
^P = Osmotic pressure, atm
Pf = P fresh water, atm
Ps = P salt solution, atm
R = R(t), density, gm(t)/L
R' = Universal Gas Constant,
S = % salt, gm(s)/100gm(t)
t = time, y
V = volume acre-feet, af
V' = flow rate, af/y
Vo = initial volume, af

W = water mass, gm(w)*af/L
w = gm(w)/L
Z = salt, gm(s)*af/L

Subscripts
in = in(flow)
ra = rain
sp = spring
ev = evaporation
se = seepage
ex = out(flow)
ot = original

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